The Dirac Equation →

The equation $H\psi = E\psi$, where

$$H = c\vec{\alpha}\cdot P + \beta mc^2$$

$$(c\vec{\alpha}\cdot P + \beta mc^2)\psi = E\psi$$

$$(c\alpha_x P_x + c\alpha_y P_y + c\alpha_z P_z + \beta mc^2)\psi = E\psi$$

with $P_x = -i\hbar \frac{\partial}{\partial x}$

To write it an more explicit form we replace α's and β by specific matrices and replace ψ by four-component column symbol.

$$\begin{bmatrix} mc^2 & 0 & cP_z & c(P_x - iP_y) \\ 0 & mc^2 & c(P_x + iP_y) & -cP_z \\ cP_z & c(P_x - iP_y) & -mc^2 & 0 \\ c(P_x + iP_y) & -cP_z & 0 & -mc^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = E \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}$$

so that this equation reduces to the four simultaneous equations

$$(mc^2)\psi_1 + cP_z\psi_3 + c(P_x - iP_y)\psi_4 = E\psi_1$$

$$(mc^2)\psi_2 + cP_z\psi_4 + c(P_x + iP_y)\psi_3 = E\psi_2$$

$$(-mc^2)\psi_3 + cP_z\psi_1 + c(P_x + iP_y)\psi_2 = E\psi_3$$

$$(-mc^2)\psi_4 + cP_z\psi_2 + c(P_x + iP_y)\psi_1 = E\psi_4$$

These equations may be expressed as.

$$(E - mc^2)\psi_1 - cP_z\psi_3 - c(P_x - iP_y)\psi_4 = 0$$

$$(E - mc^2)\psi_2 - c(P_x + iP_y)\psi_3 + cP_z\psi_4 = 0$$

$$(E + mc^2)\psi_3 - cP_z\psi_1 - c(P_x - iP_y)\psi_2 = 0$$

$$(E + mc^2)\psi_4 - c(P_x + iP_y)\psi_1 + cP_z\psi_2 = 0$$

Replace $P_x \rightarrow -i\hbar\frac{\partial}{\partial x}$ and get.

$$(E - mc^2)\psi_1 + i\hbar c\frac{\partial\psi_3}{\partial z} + i\hbar c\left[\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right]\psi_4 = 0$$

$$(E - mc^2)\psi_2 + i\hbar c\left[\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right]\psi_3 - i\hbar c\frac{\partial\psi_4}{\partial z} = 0$$

$$(E + mc^2)\psi_3 + i\hbar c\frac{\partial\psi_1}{\partial z} + i\hbar\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_2 = 0$$

$$(E + mc^2)\psi_4 + i\hbar c\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_1 - i\hbar c\frac{\partial\psi_2}{\partial z} = 0$$